Research - Outreach - Community

Atlantic Association for Research in the Mathematical Sciences

# Newsletter

Autumn 2011

# Message from the Director

On a recent grant application, I was given several choices to classify my research. One of the choices was "curiosity driven research", which led me to wonder: is there any other kind? Certainly, German mathematicians Oskar Perron and Georg Frobenius were driven by curiosity when they embarked on their research on matrices. Matrix multiplication had just been defined, and Caley and Hamilton had found the amazing connection between matrices and algebraic equations. Naturally, mathematicians at the time were eager to explore the new concepts of eigenvalues, characteristic polynomials and determinants.

I looked into the work of Perron and Frobenius because of a talk I gave to a non-mathematical audience. In my lecture, I talked about Google's algorithm for ranking search results, known as PageRank. In mathematical terms, PageRank values form the stationary distribution of a random walk on the graph formed by Web pages and hyperlinks. A patent by Larry Page, founder of Google, describes the iterative algorithm in a few paragraphs. But it is the well-known Perron-Frobenius theorem about the eigenvalues of positive matrices which gives proof that the calculation always converges.

Mathematicians know that this sort of happenstance is typical. The results of "curiosity driven research" become part of the mathematics curriculum, and find an application that their authors could not possibly have imagined. In a time where we are under pressure to operate under a very narrow definition of "applicable" research, it is good to remind ourselves and others that fortuitous connections with the past are only possible if mathematicians are allowed to follow their fancy.

That does not mean that we should limit ourselves to the single-minded pursuit of our own research. Euler, as mathematician to the Russian court, advised the tsar on gambling strategies, and even entertained the frivolous question whether it was possible to traverse all bridges of the town Konigsberg without crossing any bridge twice. Likewise, we should be ambassadors of our craft, and share our knowledge and skill with companies and colleagues, and our enthusiasm with students of all ages.

AARMS remains dedicated to mathematical research that is driven by curiosity, animated by applications, and inspired by beauty.

- Jeannette Janssen

# The Mathematics of Planet Earth 2013

2013 will be a special year of emphasis on the Mathematics of Planet Earth, interpreted as broadly as possible. Earth is a planet with dynamic processes in the mantle, oceans and atmosphere creating climate, causing natural disasters, and influencing fundamental aspects of life and life-supporting systems. In addition to these natural processes, humans have developed systems of great complexity, including economic and financial systems; the World Wide Web; frameworks for resource management, transportation, and energy production and utilization; health care delivery; and social organizations. Human activity has increased to the point where it influences the global climate, impacts the ability of the planet to feed itself and threatens the stability of these systems. Issues such as climate change, sustainability, man-made disasters, control of diseases and epidemics, management of resources, and global integration have come to the fore. Mathematics plays a key role in these and many other processes affecting Planet Earth, both as a fundamental discipline and as an essential component of multidisciplinary and interdisciplinary research.

Mathematics of Planet Earth 2013 will focus mathematical research in these fields, provide a platform to showcase the essential relevance of mathematics to planetary problems, coalesce activities currently dispersed among institutions, and create a context for mathematical and interdisciplinary developments that will be necessary in order to address a myriad of issues and meet global challenges in the future. Activities will take place around the planet. Several thematic activities will be organized jointly by more than one institute. Activities will include Thematic Programs, workshops, collaborative research groups, summer schools, activities for the public and the media, activities in K-12 Education, special Issues of Scientific Journals. Details will be announced as they become finalized.



# News

### Summer School 2012

The annual AARMS Summer School will begin its second decade on Monday July 16, 2012 at Memorial University in St. John's, Newfoundland and Labrador. This four week school is intended to attract graduate and exceptional undergraduate students from all parts of the world. The 2012 school will offer two courses in algebra and two in combinatorics. Darryn Bryant from the University of Queensland will deliver a course on Combinatorial Designs and Graph Decompositions; Pawel Pralat from Ryerson University will deliver a course on the Probabilistic Method and Random Graphs; Alberto Elduque from the University of Zaragoza, Spain, will deliver a course in Lie Theory; Nicolas Andruskiewitsch from the University of Cordoba, Argentina, and Leandro Vendramin from the University of Buenos Aires, Argentina, will deliver a course on Hopf Algebras and Applications. Each course will be a Memorial University graduate course, so we hope that students' home institutions will offer transfer credits. Certainly, we are prepared to help students achieve local credit in any way possible. The local expenses of all students (accommodation, meals, textbooks) will be met in full by the School. There are no registration fees. Organizers of the 2012 Summer School are Mikhail Kotchetov (MUN) and David Pike (MUN). www.aarms.math.ca/summer/2012/index.html

## Moving Frames

First introduced by the Estonian mathematician Martin Bartels, and primarily developed by Élie Cartan, the theory of moving frames (repères mobiles) is a powerful tool for studying geometric properties of manifolds under the action of a transformation group G. Once a geometric structure, for example a Riemannian metric or differential equation, is recast within the framework of differential 1-forms, Cartan's algorithm produces a G-invariant coframe which encapsulates all the local information of the problem. The invariance of the coframe implies that the coefficients the structure equations obtained by exterior differentiating the coframe 1-forms are (local) differential invariants. By differentiating the structure coefficients with respect to the coframe derivatives, that is the vector fields dual to the coframe 1-forms, one obtains a hierarchy of higher order differential invariants that fully characterize the geometric structure. The prototypical example taught in calculus is the Frenet–Serret frame {T, N, B} attached to a regular Euclidean curve in  $\mathbb{R}^3$ , consisting of the unit tangent, normal and binormal vectors. Computing the Frenet-Serret equations

$$\mathbf{T}' = \kappa \mathbf{N}, \qquad \mathbf{N}' = -\kappa \mathbf{T} + \tau \mathbf{B}, \qquad \mathbf{B}' = -\tau \mathbf{N}$$

we uncover two differential invariants, namely the curvature  $\kappa$  and torsion  $\tau$ , which locally completely characterize a curve.

More sophisticated applications of moving frames appear in Riemannian, Lorentzian, Finsler, conformal and parabolic geometry. They are also used to study real hypersurfaces in complex manifolds and to understand integrability and conservation laws of differential equations and variational problems. Under the guidance of Professor Milson, one of my current research projects consists of applying the moving frame apparatus to study geometrical properties of scalar third-order ordinary differential equations (ODEs) under the (pseudo-) group of contact transformations. We are particularly interested in characterizing the third-order differential equations whose Cartan structure equations admit only constant differential invariants. These differential equations are geometrically interesting as they can be given the structure of a homogeneous space (locally). But more importantly, they are at the frontier of the most computationally demanding third order ODEs to classify in the sense that high order differential invariants are required. The ultimate objective of this research project is to obtain a sharp lower bound on the order of the differential invariants required to invariantly classify all third-order ODEs under contact transformations.

Francis Valiquette, AARMS Postdoc

# Alex Kalamkarov wins 2011 CANCAM Gold Medal

Dr. Alex Kalamkarov (Professor of Mechanical Engineering) was awarded the 2011 CANCAM (Canadian Congress of Applied Mechanics) Gold Medal in recognition of his outstanding contribution to the fundamental analysis, micromechanical modelling and optimization of composite materials and smart structures. The CANCAM Medal is awarded only once in two years for the "outstanding contribution in the area of Applied Mechanics." It is first time that the CANCAM Medal is awarded to a researcher from the Atlantic Canada. Dr. Kalamkarov was awarded the Medal at the CANCAM-2011 Congress at UBC in Vancouver.



David Dr. Navlor. President of the CANCAM Central Committee awards Dr. Alex Kalamkarov the 2011 CANCAM Gold Medal at the award ceremony at the University of British Columbia on June 7th, 2011.

### In Memory: Dr. Robb Fry

by John Quinn, Joe Apaloo and Sergei Aalto

Our colleague and friend Dr. Robb Fry passed away on August 31, 2011. He was Professor of Mathematics at Thompson Rivers University, and formerly taught at the University of Northern British Columbia and St. Francis Xavier University (StFX). A graduate of Queen's University with a PhD from the University of Toronto, Robb was an active researcher in real analysis and approximation theory. His research was funded by an NSERC Discovery Grant throughout his career. To his friends at StFX he was known as someone who always welcomed a visit to his office, particularly if they had a challenging question about real analysis. Dr. Fry was a teacher whose love of mathematics was apparent and who had a real talent to convey both the meaning of mathematics and the excitement of doing mathematics to his students.

Rob was an excellent entertainer. While at StFX, he often held dinners and lunches at his home to which many work colleagues and friends were invited. He held these parties for the pure love of getting people together. In the hallway of his home department at StFX and in his office, Robb was always cheerful and welcoming. In his own words, Antigonish and StFX was the place where his marriage, fatherhood and career began. While he lived in Kamloops , British Columbia he would provide accommodation and meals for friends from Antigonish who visited at his house, and thus he maintained the friendships he had established while he lived in Antigonish. He is greatly missed by his wife Pamela, daughter Georgia, and his friends in the mathematics community.



Dr. Robb Fry

# The Collaborative Research Group in Dynamic Systems

Starting from Summer 2011, AARMS is supporting a collaborative research group in dynamical systems. The goal of this grant is to foster collaborations and train students in dynamical systems. The group's members are currently involved in projects on cosmology, swarming, nonlocal systems, and PDE's with delay.

The grant helped to support financially two undergraduate student's summer research: Yuxin Chen, working with Prof.

Theodore Kolokolnikov, and Susanna Rumsey, working with Prof. Theodore Kolokolnikov and Prof. David Iron. We also supported in part a visit by Joey Latta to the European Space Agency. Finally, we enjoyed the visits of a postdoc David Umisky and a PhD student, Hui Sun, both from UCLA. They collaborated with our group on swarming models.

Joey Latta, a Phd. student of Dr. Alan Coley's, spent three months (Apr. - Jul 2011) researching at the European Space Agency in Noordwijk, The Netherands. There he worked on the Advanced Concepts Team, which is a group of Phd and post doctoral students working on new ideas for research for the Space Agency. In particular, Joey worked with Dr. Sante Carloni on the "Gravito-Magnetism" problem, which though theoretically discovered in 1917, has only recently (Apr 2011) been tested with a reasonable degree of accuracy. Using Hamiltonian formalism, the system was recast in a manner suited to analysis using dynamical systems techniques. This investigation is ongoing.

Yuxin's summer project was on the systems of coupled oscillators with a time delay. In the limit of many oscillators, we reformulated this system as two coupled PDE's with a time delay. We then analysed the behaviour of the PDEs near the Hopf bifurcation; the long-time dynamics were found satisfy a certain Ginsburg-Landau PDE.

Susanna worked on a simple model of cancer invasion, and a related voter or Ising model. In the continuous limit the model becomes a non-local PDE (with an integral term). Its solution can exhibit complex spatio-temporal behaviour, which typically consist of travelling waves. Under certain assumptions, simple travelling-wave solutions can be studied analytically. In particular, we were able to predict under which conditions a travelling wave propagates throughout the medium, or gets "stuck" inside, leading to complex dynamics.

Insects such as locusts or mosquitoes tend to form swarms. These can be modeled either as a large system of ODE's, or a PDE with non-local terms. The study of such swarms is an ongoing and collaborative effort by our group and another group at UCLA. This summer, we had two visitors from the UCLA group (Hui Sun and David Uminsky).

Please contact us if you would like to participate in our group. We have projects for students at all levels including undergraduate.



Complex patterns in a simple aggregation model. The top row shows numerically computed equilibrium state while the bottom row shows the corresponding spherical harmonic as predicted analytically.

# Recent and Upcoming Events

#### Special session on High-Dimensional Data Analysis at APICS Conference

Organizers: Martin van Bommel Location: St Francis Xavier University, Antigonish Date: October 15-16, 2011 Contact: Martin van Bommel

#### Workshop on Numerical Weather Models for Space Geodesy Positioning

Organizers: Marcelo Santos Location: University of New Brunswick, Fredericton Date: October 18-20, 2011 Contact: Marcelo Santos

#### **Combinatorial Games Workshop**

Organizer: Richard Nowakowski Location: Dalhousie University, Halifax Date: June 22-26, 2012 Contact: Richard Nowakowski

#### Graphs & Games Student Research Weeks

Organizers: Bert Hartnell, Jeannette Janssen, Jason Brown, Richard Nowakowski Location: Dalhousie University, Halifax Date: June 22 - July 15, 2012 Contact: Richard Nowakowski

#### **Canadian Undergraduate Mathematics Conference**

Organizers: Rodney Earl, Josh Sarada, Faisal Md. Rahman, Andrea Hyde, Spencer Hunt, Crystal Parras, Jodie Foster Location: UBC Okanagan Date: July 11-15, 2012 Contact: Jodie Foster

#### International Symposium in Statistics

Organizer: Brajendra Sutradhar Location: Memorial University, St. John's Date: July 16-18,2012 Contact: Brajendra Sutradhar

#### **AARMS Summer School**

Organizers: Mikhail Kotchetov and David Pike Location: Memorial University, St. John's Date: July 16 - Aug 10, 2012 Contact: Mikhail Kotchetov and David Pike

#### Two Weeks at Waterloo - A Summer School for Women in Math

Organizers: Barbara Csima and Kathryn Hare Location: University of Waterloo Date: August 12-25, 2012 Contact: Kathryn Hare

#### 24th Canadian Conference on Computational Geometry

Organizers: Greg Aloupis and David Bremner Location: Memorial University, Saint John's Date: August, 2012 Contact: Greg Aloupis

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As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality. ~Albert Einstein, Sidelights on Relativity