

Dr. Richard J. Nowakowski Department of Mathematics & Statistics Room 323, Chase Bldg Halifax, Nova Scotia Canada B3H 3J5 Telephone: (902)-494-6635 Fax: (902)-494-5130

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#### Report: Games@Dal 2016

### Overview:

The event was held August 10-13, 2016 and organized by Melissa Huggan, Svenja Huntemann, Urban Larsson and myself. Talks were presented on the first day. The other days were spent in groups working on problems. Fifteen people, including four students, from North America and Portugal attended the Talks and Workshops. Another six people from Dalhousie attended the talks or parts of the Workshops.

For the Workshops, several problems were identified in advance and the participants divided into groups and worked on these problems. Participants were free, and encouraged, to wander between groups. Before lunch and at the end of each day, progress on each problem was discussed.

The Games@Dal Workshops differ from most other meetings in that it brings a diverse group of experts and students together to work on various aspects of important research topics in a free-flowing atmosphere. At the same time, students are also able to work with these top researchers and participate and contribute to research. The Workshop is very popular and the award from AARMS makes it all possible. (I have already received enquiries about the next Workshop from new potential attendees in Israel, France and USA.)

From the Workshop, there are four papers in preparation, and each student is a co-author. Specifically, the papers are:

M. Fisher, M. Huggan, S. Huntemann, Split-ends Nim;

- T. Khovanova, R. Nowakowski, Nim on Ideals;
- K. Burke, T. Khovanova, R. Nowakowski, A. Rowland, C. Tennhouse, *Hiding counterfeit coins.*
- A. Carvalho, N. McKay, R, Nowakowski, C. Santos, *Short Disjunctive Sum: a new approach.*

### **Participants:**

Canada Melissa Huggan\* Svenja Huntemann\* Urban Larsson Neil McKay\* Rebecca Milley Richard Nowakowski Israel Rocha USA Portugal Kyle Burke Alda Carvalho Mike Fisher Carlos Santos Tanya Khovanova Amelia Rowland\* Simon Rubinstein-Salzedo Craig Tennenhouse

\* = student

# Talks: (August 10)

## Neil McKay: Hereditary Transitive Games

Playing in a transitive game a player can reach in one move any position that they can reach in two or more consecutive moves. A position is hereditarily transitive if all its sub-positions are transitive. Hereditarily transitive games were introduced by Ottaway and Nowakowski under the name option-closed.

We show that we can carefully apply domination and reversibility while preserving the hereditarily transitive structure. For each position we associate a unique simplest hereditarily transitive position. We also discuss values of hereditarily transitive positions, which include numbers, nimbers, hot games, and many dicotic values.

## Craig Tennenhouse: PenFib Nim, a new conjoined game

Two combinatorial game rulesets A and B can be combined to form a conjoined game, wherein A is played to completion and then its terminal position becomes the initial position of a game following ruleset B. In this talk we examine an impartial conjoined game composed of a nim variant we call Penultimate Nim conjoined with the single-heap subtraction game Fibonacci Nim, which formed a recent undergraduate research project. Outcome classes of some small positions are presented and possible extensions are discussed.

# Carlos Santos with Richard Nowakowski & Alexandre Silva: *3-Player NIM with the `Podium Rule'*

In the first part of the talk, we briefly discuss the state of art about the research on combinatorial 3-player games. Playing with the «podium rule convention», if a player cannot be last, he tries to be last but one. In his paper "N-person Nim and N-person Moore's games", S Liu presented a very elegant characterization of the P-positions of 3-player NIM with podium rule (and, in general, for N-player NIM). In the second part of the talk, we show how Liu's work can be extended, presenting the general reduction process and related canonical forms.

## Israel Rocha with Urban Larsson: Eternal Picaria:

This game could be called 3 Men's Morris. The authors show that with best play the game is a tie. Several open questions remain.

Alda Carvalho with Carlos Santos: *Some Notes on Disjunctive Short Sum* About disjunctive short sum (dss), John Conway wrote the following: «The compound game ends as soon as any one of the component games has ended» (ONAG, 1976). But, what does «any one of the component games has ended» mean?! The discussion about that question leads to, at least, four different definitions! Paul Ottaway, for misère-play, in his Phd thesis (2009), studied one of the possibilities. We will discuss and extend some of his interesting ideas. We will present some surprising facts as the following: a) dicotic misère with dss is partizan normal-play; b) dss is normal-play with «race games»; c) dss presents a distributive lattice structure very similar to normal-play; d) Essentially, we compare in the dss structure as we compare in normal-play; e) The only reason why dss is not normal-play is because the existence of «race games» and non invertible elements; f) We have domination, standard reversibility, atomic reversibility and canonical forms; g) impartial with dss is impartial normal-play. As an example , we present the ruleset COLOR NIM, some related results (for 3 piles), open problems and conjectures.

# Urban Larsson: *Hopeful windows and fractals in cellular automata and combinatorial games*

Hopeful windows and fractals in cellular automata and combinatorial games This talk concerns 2-player impartial combinatorial games, for which the outcomes correspond to updates of cellular automata (CA), a class that generalizes Wolfram's elementary rule 60 and rule 110 (Cook 2004). The games extend the triangle placing games (Larsson 2013) by: at each stage of the game the previous player may block at most a fixed number of options in a certain hopeful window suggested by the next player. Thus, there is a short discussion phase before each move, and in this application, it reduces the number of options for the next player. We also study fractals and partial convergence in a subclass of the CA. Open questions: find other classes of games with such "move discussions". Study the updates of the CA, and discuss complexity problems; is there a full class of un-decidable games, perhaps related to rule 110? We know already that Pascal's triangle modulo 2 (similar to rule 60) reappears in different shapes and formations for various blocking maneuvers. This suggests some further classification. The fractal behavior, in special classes, is guite astonishing, because it is not given directly by the update rules, but yet can be explained rigorously.

### Tanya Khovanova: Cookie Monster Game

Cookie Monster likes cookies. His mommy used his love for cookies to teach him to think and to play some mathematical games. She set up the following system with cookies. Cookie Monster's Mommy has a set of k jars filled with cookies. In one move she allows Cookie Monster to choose any subset of jars and take the same number of cookies from each of those jars. Cookie Monster always wants to empty all of the jars in as few moves as possible.

## **Expenses:**

Coffee Break:\$148.871st day of WorkshopResidence rooms:\$810.00For Khovanova, Milley, RowlandTravel and\$2899.84For Drs Carvalho and Santos (Aug 9-26)\*Total:\$3858.71

Amount awarded \$3000 by AARMS

Dr. Nowakowski's NSERC Discovery Grant covered the remaining \$858.71.

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Dr. Richard J. Nowakowski

\* Drs. Carvalho and Santos stayed for further collaboration with Fisher, Huggan, Huntemann, Larsson, and Nowakowski, which is part of the original request.