Atelier conjoint AARMS-CRM: Progrès récents sur les équations différentielles fonctionnelles et avec retard

Joint AARMS-CRM Workshop: Recent Advances in Functional and Delay Differential Equations

Location
Dalhousie University & Lord Nelson Hotel, Halifax

Date
2007-11-01 - 2007-11-05

Organisers
Brunner, Hermann
Humphries, A.R.
Pelinovsky, Dmitri

Local Organisers
Keast, Pat
Muir, Paul

Conférenciers / Speakers
Adhikari, Mohit
Belair, Jacques
Bellen, Alfredo
Breda, Dimitri
Brunner, Hermann
Buono, Luciano
Campbell, Sue Ann
De Luca, Jayme
Elmer, Christopher E.
Enright, Wayne
Guglielmi, Nicola
Gyori, Istvan
Hartung, Ferenc
Humphries, Antony R.
Iooss, Gérard
Krisztin, Tibor
Lajoie, Guillaume
Lessard, Jean-Phillipe
Ma, Jianfu
Mackey, Michael C.
Mallet-Paret, John
Maset, Stefano
Ou, Chunhua
Palencia de Lara, Cesar

Panayotaros, Panayotis
Pedas, Arvet
Pelinovsky, Dmitri
Rost, Gergely
Rothos, Vassilis
Susanto, Hadi
Vainchtein, Anna
Van Vleck, Erik S.
Vermiglio, Rossana
Wall, David J.N.
Walther, Hans-Otto
Wu, Jianhong
Zhao, Xiaoqiang
Zou, Xingfu F.

Nombre de participants / Number of participants
45

Resumé de l'activite / Activity resume
Delay differential equations arise in many applications, and in the case of constant delays a fairly mature theory of such problems as infinite dimensional dynamical systems has been developed. However, models in physical and biological applications are increasingly encompassing features which do not fit this theory, often having non-constant and state-dependent delays. Mixed type differential equations with both advanced and retarded arguments also arise,for example as the defining equations for travelling waves in nonlinear lattices. Volterra functional (integral and integro-differential) equations with variable and state-dependent delays, are also applied with increasing frequency. The theory of such problems is still far from complete, though significant progress is being made. A large gap also exists in the numerical analysis and computational solution of such functional equations. So the aim of this workshop was to bring together researchers and students working on such equations equations from applied, numerical and theoretical viewpoints, to give a wide perspective on recent results, current research and open problems, in these overlapping fields. There were five plenary speakers;

Gerard Iooss (Université de Nice, France)
``Localized waves in lattices of Fermi-pasta-Ulam type"

Cesar Palencia (Universidad de Valladolid, Spain)
``Runge--Kutta convolution quadrature methods for linear homogeneous Volterra equations"

Nicola Guglielmi (Universita degli Studi di L'Aquila, Italy)
``Numerics of delay differential equations"

Tibor Krisztin (University of Szeged, Hungary)
``Differential equations with state-dependent delay"

Jianhong Wu (York University)
``Neural computation with periodic attractors: memory and time lags"

but all participants were invited to give a talk, and nearly all did, which necessitated parallel sessions one afternoon.

One theme that ran through the workshop were the use of delay equations in applications, particularly biological applications. Here it would be natural for the delays to be state dependent, but most models treat delays as constant, mainly because of the lack of techniques and theory for the state dependent problems. It is clear that provided mathematicians can supply the theory and methods to solve the problems, biological applications will provide a plentiful source of new state-dependent delay problems.

The growth in the interest in state dependent problems, between the ``Function Differential Equations" workshop held at Banff in 2004 and now was striking. From a theoretical viewpoint, existence results for periodic solutions, in particular slowly oscillating periodic solutions (with period larger than twice the delay), were presented under a variety of situations, including negative and positive feedback problems. Numerical methods are also under development for state-dependent problems, where the problem of solution termination for neutral problems has come under investigation. In these problems there is no smoothing of the initial function, and break points where the solution has discontinuous derivative arise at solution dependent time values that cannot be precomputed, where the solution may terminate. Methods were shown to continue solutions beyond termination in the second scenario using either a Filipov-like set-valued extension of the differential equation, or by regularisation. The relevance of these methods was shown in an example where a particular initial vale problem passed through several terminations before converging to an attractive periodic orbit in a region of phase space without terminations. Another example of recent complementary developments in theory and numerics concerns stability of fixed points, where efficient methods for the numerical computation of characteristic values were presented alongside the theory being developed to link nonlinear and linear stability of fixed points for state-dependent problems (which is a much harder problem than in constant delay equations).

The talks on Volterra integral, integro-differential and more general functional equations illuminated, on the one hand, the state of the art of the numerical treatment of such equations. In particular, a general approach based on the abstract representation of the numerical solution to a Volterra equations (of parabolic or non-parabolic type) in terms of the analytical solution leads to a comprehensive numerical stability analysis. In the case of Volterra integral or integro-differential equations with weakly singular kernels it appears advantageous to subject them to a 'smoothing tranformation'; the resulting improved regularity of the solutions leads to more efficient numerical methods (in particular, collocation methods). On the other hand, many numerical issues remain to resolved. These include the analysis of efficient methods for Volterra equations with highly oscillatory kernels, and the design of reliable numerical codes for Volterra type functional differential and integral equations especially for problems with state-dependent delays).

Another area of considerable interest, were nonlinear travelling wave problems on lattices, arising from materials science, atomic physics and nonlinear optics. These problems in different applications lead to dissipative or Hamiltonian advanced-retarded equations on the entire axis with homoclinic, heteroclinic and periodic solutions. In the Hamiltonian case, recent progress has been made using methods of center manifold reductions and normal form transformations. Because homoclinic and heteroclinic orbits originate from a bifurcation of multiple zero eigenvalues in the presence of non-zero purely imaginary eigenvalues, existence of such wave solutions is a subject to a set of constraints. Melnikov integrals determine conditions of non-trivial bifurcations, and numerical evaluation of Melnikov integrals (also related to Stokes constants in beyond-all-order asymptotic expansions) is a subject of much study in the area. Other talks were devoted to variational characterization of periodic travelling waves in lattices and time-periodic forcing of localized waves due to diffraction management in Hamiltonian lattices.

If the original physical system is governed by dissipative reaction-diffusion equations, nonlinear traveling waves become solutions of dissipative advanced-delay equations. Several talks were devoted to propagation failure for front (heteroclinic) solution in a scalar nonlinear heat equation. This phenomenon has been investigated recently in many details.

Many other problems and results were presented, but I will mention just two more, partial differential equations with delays, on which limited work has been done as yet, where there is potential for significant future attention, and the Wheeler-Feynman problem of classical electrodynamics, which might not have the potential to be the next big trendy research area, but is certainly a fiendishly difficult and interesting implicitly state-dependent neutral advanced-delay differential equation.

The meeting was jointly organised by CRM and AARMS and took place in Halifax, initially at Dalhousie University. As well as a strong turnout of the Canadian delay equations community, many participants travelled from Europe, and some came from as far afield as Brazil, New Zealand and Estonia. It was great success at getting researchers from different fields and countries to interact; in the freetime there were always several discussions going on near the lecture room. One unwelcome guest was Hurricane Noel, whose remnants swept across Halifax on Saturday night, soaking everyone walking back to the hotel after the seminars. Worse still, Sunday morning we discovered Dalhousie was in one of the powerless parts of the city, and we had lost our venue. So, we all owe a big vote of thanks to the local organiser Pat Keast, who saved the conference arranging an alternative venue at the Lord Nelson Hotel on Sunday morning, at less than an hours notice. In case you were wondering, the CRM does not have Hurricane insurance! In all, it was a very memorable workshop.

Total total cost was $40,122.50 of which AARMS provided $10,000 and registration fees $1500, the balance coming from CRM.