The 2nd Atlantic Analysis Days

- in Honour of Tony Thompson -

On March 30 and 31, 2007 the Department of Mathematics and Statistics of Dalhousie University hosted a conference, sponsored by AARMS, in honour of Tony Thompson on the occasion of his 70th birthday. The focus of the conference was on Convex Geometry, and two one-hour talks were given by leading experts in the field: Rolf Schneider of the University of Freiburg in Germany spoke on "Stability results in convex geometry", and Juan Carlos \'Alvarez Paiva spoke on "The Holmes-Thompson volume in normed and Finsler spaces".

A further 13 half-hour talks were presented by participants from Dalhousie, the Atlantic region, and abroad. One of them was by Tony's former Ph.D. student Rolf Clackdoyle who was unable to attend; his talk was presented by the conference organizer. The speakers from Dalhousie were Jonathan Borwein, Georg Hofmann, S. Swaminathan, and Keith Taylor. One family of three attended the conference: Art Finbow, his son Stephen Finbow, and his daughter Wendy Finbow-Singh, all mathematics Ph.D.s and university faculty who had all been taught by Tony. Wendy, whose M.Sc. thesis had been co-supervised by Tony, also gave a talk at the conference.

A conference dinner with more than 40 guests took place on the Friday evening, March 30.

Further information on this event, including a list of speakers and titles, can be found at the conference website, http://www.aarms.math.ca/events/atlantic/

- Karl Dilcher

Abstracts

Juan Carlos Álvarez Paiva, Université des Sciences et Technologies de Lille
The Holmes-Thompson volume in normed and Finsler spaces

The Holmes-Thompson volume on an n-dimensional normed space X is the multiple of the Lebesgue measure for which the volume of the unit ball of X equals its volume-product divided by the volume of the Euclidean n-dimensional unit ball. This talk will survey the wealth of interactions between convex, integral, symplectic, and metric geometry which this seemingly simple definition has uncovered.

Gautier Berck, Scuola Normale Superiore, Pisa
A solution to Busemann-Petty tenth problem

We will give a solution to Busemann-Petty tenth problem by presenting the ideas of the proof that Busemann's density is totally convex. As nice consequence for the calculus of variations, this implies the minimality of affine k-dimensional discs for the Haussdorff measure in finite dimensional normed spaces.

Jonathan Borwein, Dalhousie University
Three results Tony did/would/should like

Abstract: "The title should suffice."

Robert Dawson, St. Mary's University
Some results on Čebyšev sets in hyperspaces over a Minkowski space

A set X in a metric space has the *Čebyšev property* if every point in the space has a unique nearest point in X. In Euclidean spaces this is equivalent to being closed, convex, and nonempty; but in some other metric spaces convexity neither implies the Cebysev property (even among closed nonempty sets) nor is implied by it. The situation is particularly problematic for *hyperspaces*.

By a hyperspace we will understand some collection of subsets of a metric space, with a metric derived from that of the original space; a typical example is $\mathcal{K}^n$, the hyperspace of nonempty compact convex sets of R^n with the Hausdorff metric. In this space, and in spaces related to it, there are several known types of Čebyšev set, apparently unrelated. Some are infinite-dimensional and strictly convex, some low-dimensional and deriving the Čebyšev property from the linear structure of the hyperspace, and some have the Čebyšev property for purely order-theoretic reasons.

In this talk, based on work with A.Bogdewicz and M. Moszynska, we examine some of the different types of Čebyšev set in hyperspaces and some possibilities for a more unified classification.

(Joint work with Agnieszka Bogdewicz and Maria Moszynska.)

Wendy Finbow-Singh, St. Mary's University
Generic rigidity of spheres with holes and blocks

From the work of Cauchy, Dehn and Alexandrov, it is known that convex triangulated spherical polyhedra are first-order rigid and therefore the graphs are generically rigid in 3-space, built as bar and joint frameworks. If we shift a few of these bars, making some regions into holes with no bars, and bracing some other regions up to be smaller rigid blocks, the question arises: when are the modified frameworks still first-order rigid?

A few examples were studied about 20 years ago, as the method of 'vertex splitting' was just being introduced as a method for proving generic rigidity. There were some conjectures of larger patterns which should be derivable from simple base structures by vertex splitting. The key is to detect, inside a larger framework an induced 'core set' from which it can be constructed by vertex splitting. We will refine and prove these conjectures, giving a broad method for verifying which patterns of modified triangulated spheres, with holes and blocks, will be generically rigid.

The sample patterns that can be created with these methods offer good examples for illustrating the possibilities of flexibility and rigidity communication between distant portions of a framework. As such, they offer simple 'toy models' for 'allostery' (shape change at a distance) in proteins and other biomolecules viewed as mechanical frameworks.

Georg Hofmann, Dalhousie University
The geometry of groups presented by generators and relations

Presenting a group by generators and relations is a very efficient way to specify a group algebraically. But it can be difficult to determine properties of a group defined in this way, for instance to decide whether or not the group is finite, or to solve the word problem of the given presentation of the group, i.e. the algorithmic problem of deciding, given as input two words in the generators, whether they represent the same group element. In many cases, geometric interpretations of the group help understand these properties. One of the most studied examples is the class of Coxeter groups. I propose to indicate possibilities of generalizing results about this class to a larger one.

Shafiqul Islam, UPEI
Invariant measures and Frobenius-Perron operator of random dynamical systems

Invariant measures of dynamical systems describe long time behavior of the systems and play an important role in understanding their chaotic nature. We discuss properties of Frobenius-Perron operator of dynamical systems and show that the Frobenius-Perron operator is a tool for proving the existence of invariant measures of the systems. We introduce random maps and their Frobenius-Perron operators. We show a few results about the existence of invariant measure for random maps.

Daniel Klain, University of Massachusetts at Lowell
An easy error estimate for the isoperimetric deficit

A four part dissection and rearrangement provides an especially simple proof of the isoperimetric inequality in the plane as well as a new approach to Bonnesen-type error estimates for the isoperimetric deficit of compact convex sets and of star bodies that are centrally symmetric with respect to the origin.

Monika Ludwig, Polytechnic University New York
Projection and intersection bodies

Operators that map convex bodies to convex or star bodies and are $\mathrm{SL}(n)$ covariant play an important role in the geometry of convex bodies. Examples are obtained by projection and intersections bodies. Valuations allow us to characterize these operators.

In the talk, we describe some old and new characterization theorems as well as important applications of projection and intersection bodies.

Rolf Schneider, Universität Freiburg
Stability results in convex geometry

One might say that a geometric inequality with known extremal cases, or a geometric uniqueness theorem of any kind, is only fully understood if one is able to demonstrate that small changes of the conditions that enforce uniqueness lead to small, computable changes of the outcome. This is the idea of geometric stability. We give a survey over some classical uniqueness theorems from convex geometry where stability has been established (sooner or later), and we describe some new developments. The old and new results come from different areas: isoperimetric inequalities and applications, balls and ellipsoids, simplices, geometric tomography.

S. Swaminathan, Dalhousie University
Isometries on metric spaces

The talk is about the question 'When is a mapping of a metric space into itself, that preserves unit distances, an isometry?'

Keith Taylor, Dalhousie University
Smooth points in von Neumann algebras

A von Neumann algebra is a C^*-algebra that is isometrically isomorphic to the dual of a Banach space; thus $L^\infty(\mu)$ and $\mathcal{B}(\mathcal{H})$ are examples of von Neumann algebras. For a point x in a normed space X, a support functional is a continuous linear functional of norm 1 on X whose value at x is $||x||$. If there is only one support functional, x is called a smooth point in X. Building on the known characterizations of smooth points in L^∞ spaces and the space of bounded operators on a Hilbert space, smooth points in an arbitrary von Neumann algebra will be described. This is based on joint work with Wend Werner.

Alan Thompson, MDA, Vancouver
Radon transforms and radar remote sensing

Radon transforms are integral transforms that involve integrating functions over submanifolds. For example, the classical Radon transform of a function on Euclidean space is a function on hyperplanes obtained by integrating the original function over hyperplanes. The connection between Radon transforms and medical imaging is well-known. Not so well-known is the connection between Radon transforms and radar imaging. In this talk, an overview of Radon transforms, radar imaging, and their connections will be given. Relationships between Radon inversion formulas and radar data processing algorithms will be explored. For example, the projection-slice theorem for Radon transforms provides a derivation of what is known as the polar format algorithm in spotlight mode synthetic aperture radar.

Jie Xiao, Memorial University
Sharp sobolev and isoperimetric inequalities split

This talk shows that each of the sharp (endpoint) Sobolev inequality and the isoperimetric inequality can be split into two sharp and stronger inequalities through either the 1-variational capacity or the 1-integral affine surface area.