

**2013 AARMS Workshop on Analytic Spaces and Their Operators**  
**Report to the AARMS Board of Directors**

2013 AARMS Workshop on Analytic Spaces and Their Operators took place at Memorial University in St. John's from July 9, 2013 to July 12, 2013. More details about the AARMS events programme are available at

<http://www.aarms.math.ca/events/index.html>

But, for more information on the 2013 AARMS Workshop on Analytic Spaces and Their Operators, please see

<http://www.aarms.math.ca/events/AnalyticSpaces.pdf>

and Appendix A – Schedule of Workshop on Analytic Spaces and Their Operators. Each of 13 invited talks had 45 minutes in total - and the talks represented fast recent developments ranging from aspects of analytic spaces, over studies of some special function-theoretic operators up to the study of moduli spaces of holomorphic bundles as well as convex geometry. Both speakers and listeners felt this very demanding and interesting.

Altogether, 31 participants enrolled in this event. Among them there were: 7 participants from China and USA; 3 participants from Quebec; 21 participants from Newfoundland. In addition, several people (from USA, Spain, Finland, China) including Kehe Zhu were unable to come due to some reasons. A list of attendants and their home universities appears in Appendix B – Participants of Workshop on Analytic Spaces and Their Operators.

Due to a small amount of funding from AARMS and Jie Xiao's Start-up and URP of Memorial University, we did not charge all participants for registration, but did cover residence cost of each participant outside St. John's. One participant from Quebec received a round trip air-ticket. Six participants from China and Canada got meals support. Meanwhile, we provided free coffee, computing and wireless services. A summary of our budget is provided in Appendix C – Budget Summary of Workshop on Analytic Spaces and Their Operators. Of course, we are most grateful to AARMS and the sponsors from the bulk of our funding. Last but not least, we would like to thank the staff of the Dept. of Math. & Stat. at Memorial University for their warm support.

Some social activities including a welcome-reception and a half-day excursion complemented our workshop.

Jie Xiao and Kehe Zhu  
Organizers of Workshop on Analytic Spaces and Their Operators  
August 14, 2013

## Appendix A – Schedule of Workshop on Analytic Spaces and Their Operators

**July 9, 9:00 - 13:30:** This is a registration time. The registration office is HH 3003 (the General Office) of Henrietta Harvey (HH, i.e., Math) Building. A welcome coffee (**July 9, 13:30-14:00**) will be provided in HH3022 of Math Building.

**July 9, 14:00 - 16:00; July 10-11, 9:00 - 16:00:** All talks will be 45 minutes in length, and be given in HH 3017 of Math Building; A coffee break (15 minutes) between any two talks in the morning or afternoon will take place in HH3022 of Math Building. Here is the timetable of speakers each of whom will chair one's follow-up talk.

Date	9:00-45	10:00-45	11:00-45	14:00-45	15:00-45	16:00-45
7.9				P. Gauthier	Z. Li	
7.10	J. Mashreghi	T. Ransford	J. Wang	B. Wick	N. Zhang	
7.11	D. Ye	W. Xu/J.Z.	J. Zhou/B.Z.	T. Baird	S. Sadov	J. Xiao

**July 12, 9:00 - 16:00:** This is participant's free time - a tour to St. John's including Signal Hill and Cape Spear.

### Listed below are titles, speakers and abstracts of the talks

- Title: Moduli Spaces of Vector Bundles over a Real Curve

Speaker: T. Baird (Memorial University of Newfoundland, Canada)

Abstract: Moduli spaces of holomorphic bundles over a complex projective curve have been an important object of study in mathematics for more than 50 years. In a highly influential paper from the 80s, Atiyah and Bott used Morse theory and the Yang-Mills functional to compute the rational Betti numbers of these moduli spaces. More recently, the moduli space of vector bundles over a real curve has garnered a great deal of interest. I will define these moduli spaces, and explain how to adapt the Atiyah-Bott method to compute their Betti numbers in characteristic 2.

- Title: Holomorphic Extension-Interpolation

Speaker: P. Gauthier (University of Montreal, Canada)

Abstract: We discuss the holomorphic extension of a function from an arbitrary subset  $E$  of the complex plane  $\mathbb{C}$  to a neighborhood of  $E$ . We may replace  $\mathbb{C}$  by an arbitrary one-dimensional complex analytic set and, in particular, an arbitrary open Riemann surface.

- Title: On Coefficients of  $\lambda$ -Analytic Functions

Speaker: Z. Li (Capital Normal University, China)

Abstract: Harmonic analysis on the unit disk  $\mathbf{D}$  associated with the operators  $T_z f(z) = \partial_z f + \lambda(f(z) - f(\bar{z}))/z - \bar{z}$  and  $T_{\bar{z}} f(z) = \partial_{\bar{z}} f - \lambda(f(z) - f(\bar{z}))/z - \bar{z}$  was developed in [J. Funct. Anal. 265(2013), 687-742]. The connected base is the Dunkl-Gegenbauer system  $\{\phi_n^\lambda(z), z\phi_{n-1}^\lambda(z)\}$ , which is orthogonal on the circumference  $\mathbf{S}^1 = \partial\mathbf{D}$  with respect to the weight  $|\sin \theta|^{2\lambda}$ , and is consistent with  $\{z^n, \bar{z}^n\}$  as  $\lambda \rightarrow 0^+$ , where, up to some constants,  $\phi_n^\lambda(re^{i\theta}) = r^n \left( \frac{n+2\lambda}{2\lambda} P_n^\lambda(\cos \theta) + iP_{n-1}^{\lambda+1}(\cos \theta) \sin \theta \right)$ , and  $P_n^\lambda$ 's are the Gegenbauer polynomials. In this talk I shall present several results in another direction about the topic, that are estimates of coefficients for some classes of  $\lambda$ -analytic functions. Part of them motivates a conjecture about the univalent  $\lambda$ -analytic functions, as an analogue of the de Branges theorem (Bieberbach's conjecture).

- Speaker: J. Mashreghi (Laval University, Canada)

Title: An Application of Entire Functions of Exponential Type in Banach Algebra

Abstract: Theory of entire functions, as a well-established branch of mathematical analysis, has several applications in other domains of mathematics. In particular, this theory has been repeatedly exploited in Banach algebras. Besides mentioning some of these applications, we also illustrate a technique to prove a result about power growth in Banach algebras. More precisely, if  $a$  is an element of a unital Banach algebra,  $m \geq 0$  and  $\alpha \in (0, 1)$ , then

$$\|a^m((1+a)^n - (1-a)^n)\| = O(e^{\epsilon n^\alpha}) \quad (n \rightarrow \infty)$$

for all  $\epsilon > 0$  if and only if  $\lim_{n \rightarrow \infty} n^{1/\alpha-1} \|a^n\|^{1/n} = 0$ .

- Speaker: T. Ransford (Laval University, Canada)

Title: One-box Conditions for Carleson Measures for the Dirichlet Space

Abstract: A measure  $\mu$  on the unit disk is called a Carleson measure for the Dirichlet space  $\mathcal{D}$  if  $\mathcal{D} \subset L^2(\mu)$ . Several characterizations of such measures are known, none of them simple. In this talk, I shall discuss a family of simple sufficient conditions for  $\mu$  to be a Carleson measure for  $\mathcal{D}$ , in the spirit of Carleson's original characterization of Carleson measures for the Hardy spaces. (Joint work with Omar El-Fallah, Karim Kellay and Javad Mashreghi.)

- Speaker: S. Sadov (Memorial University of Newfoundland, Canada)

Title: Carleson Measures and Extensions of Some Classical Inequalities

Abstract: It is observed that due to Carleson's harmonic extension theorem some classical inequalities, such as those of Hilbert and Hausdorff-Young can be stated in a stronger "maximal" form. For instance, if  $f \in L^p(\mathbf{R})$  ( $p > 1$ ) and  $g = Hf$  is its conjugate function, then the function  $g^*(r) = \sup_{0 < \theta < \pi} g(re^{i\theta})$  belongs to  $L^p(\mathbf{R}_+)$ . Abstracting the essence of a proof, we are led to a measure-theoretic construction involving the notions of conditional essential supremum and conditional Carleson measure.

- Speaker: J. Wang (Zhejiang Normal University, China and Memorial University of Newfoundland, Canada)

Title: The Generalized Schwarz-Pick Estimates of Arbitrary Order on the Unit Polydisk

Abstract: Let  $\Omega$  be a homogeneous circular convex domain (a bounded symmetric domain) in  $C^N$  containing the origin. The generalization of the Schwarz-Pick estimates of partial derivatives of arbitrary order are established for holomorphic mappings from the unit polydisk  $D^n$  to  $\Omega$  associated with the Caratheodory metric. In particular, when  $\Omega = B^N$  =the unit ball of  $C^N$ , the Schwarz-Pick estimates of high order reduces to that of Y. Liu and Z. Chen.

- Speaker: B. Wick (Georgia Institute of Technology, USA)

Title: Composition of Haar Paraproducts

Abstract: Paraproducts are important operators in harmonic analysis and there are well known characterizations of when an individual paraproduct is bounded. An interesting question is to characterize when the composition of two, potentially unbounded, paraproducts have a bounded composition. In this talk we will give necessary and sufficient conditions that characterize when the composition of certain compositions of Haar paraproducts are bounded.

- Speaker: W. Xu/J.Z. (Southwest University, China)

Title: Some Convex Bodies of Constant Width in the Euclidean Space

Abstract: We construct three-dimensional convex bodies of constant width by rotating these two-dimensional via some planar convex bodies of constant width with axis of symmetry. Then we confirm The Campi-Colesanti-Gronchi Theorem that assert that among rotational three-dimensional bodies of constant width, the ball has the maximum volume and the rotated Reuleaux triangle has the least volume.

- Speaker: D. Ye (Memorial University of Newfoundland, Canada)

Title: On the Phase Transitions of Random States

Abstract: The phenomenon of quantum entanglement, first discovered by Einstein-Podolsky-Rosen in 1935, is fundamental in quantum information theory. Such phenomenon is the key ingredient for such as quantum algorithms (for instance the famous Shor's algorithm for integer factorization). Thus, the problem of detecting quantum entanglement is a central (and urgent) problem in quantum information theory. Unfortunately, this problem has been proved as a NP-hard problem.

- Speaker: N. Zhang (Memorial University of Newfoundland, Canada)

Title: Isocapacity Estimates for Hessian Operators

Abstract: In this talk, we firstly introduce a special capacity from k-Hessian equation. Then, we discover that the isocapacity estimates for k-Hessian capacity are exactly the geometrical capacity forms of Chou-Wangs Sobolev type inequality and Tian-Wangs Moser-Trudinger type inequality for the Hessian operators. This is a joint work with J. Xiao.

- Speaker: J. Zhou/B.Z. (Southwest University, China)

Title: Affine Isoperimetric Inequalities for  $L_p$  Geominimal Surface Area

Abstract: We present some  $L_p$  affine isoperimetric inequalities for  $L_p$  geominimal surface area. In particular, we obtain an analogue of Blaschke-Santaló inequality and a cyclic inequality for  $L_p$  geominimal surface areas. We give the concept of  $L_p$  mixed geominimal surface area which is a natural extension of  $L_p$  geominimal surface area and some Lutwaks results are extended.

- Speaker: J. Xiao (Memorial University of Newfoundland, Canada)

Title: Holomorphic Campanato-Morrey Spaces

Abstract: This talk will discuss description, composition, multiplication and approximation of the so-called holomorphic Campanato-Morrey spaces on the unit disk of the finite complex plane.

## Appendix B – Participants of Workshop on Analytic Spaces and Their Operators

Name	E-mail	Institution
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Note: (i) Each person outside MUN needs a wireless account and an access to the math-stat department lab computers; (ii) I:=International/ N:=National/ R:=Regional; (iii) Total participants (31) = I-participants (7) + N-participants (3) + R-participants (21).

## Appendix C – Budget Summary of Workshop on Analytic Spaces and Their Operators

Note: the final AARMS contribution (\$2000) and J. Xiao's Startup & URP contribution (\$3254.27) were adjusted to leave a balance of \$0 in our operating account at Memorial University.

Items	Amount	Remark
<b>Speakers</b>		
Hotel	\$2675.08	He-Xu-Zhu-Zhou-Ga.-Li(\$693.26); M(\$849.36); R(\$566.24); W(\$566.22)
Travel	\$758.52	Gauthier's air-ticket+taxi
Meals	\$1155	Li(\$275); Gauthier(\$220); He(\$165); Xu(\$165); Zhou(\$165); Zhu(\$165)
<b>Hospitality</b>		
Coffee/Food	\$340.50	9 times
Excursion	\$ 0	
Lunch	\$299.41	8 speakers +1 organizer+3 participants
<b>Total</b>	\$5228.51	AARMS(\$2000 for residence); Xiao's Startup&URP (\$3228.51 for rest)