

Summer Graduate Schools 2019

Commutative Algebra and its Interaction with Algebraic Geometry

Dates: June 03 – June 14, 2019

Location: University of Notre Dame

Organizers: Craig Huneke (University of Virginia), Sonja Mapes (University of Notre Dame), Juan Migliore (University of Notre Dame), Claudia Polini (University of Notre Dame), Claudiu Raicu (University of Notre Dame)

Description

Linkage is a method for classifying ideals in local rings. Residual intersections is a generalization of linkage to the case where the two 'linked' ideals need not have the same codimension. Residual intersections are ubiquitous: they play an important role in the study of blowups, branch and multiple point loci, secant varieties, and Gauss images; they appear naturally in intersection theory; and they have close connections with integral closures of ideals.

Commutative algebraists have long used the Frobenius or p -th power map to study commutative rings containing a finite field. The theory of tight closure and test ideals has widespread applications to the study of symbolic powers and to Briançon-Skoda type theorems for equi-characteristic rings.

Numerical conditions for the integral dependence of ideals and modules have a wealth of applications, not the least of which is in equisingularity theory. There is a long history of generalized criteria for integral dependence of ideals and modules based on variants of the Hilbert-Samuel and the Buchsbaum-Rim multiplicity that still require some remnants of finite length assumptions.

The Rees ring and the special fiber ring of an ideal arise in the process of blowing up a variety along a subvariety. Rees rings and special fiber rings also describe, respectively, the graphs and the images of rational maps between projective spaces. A difficult open problem in commutative algebra, algebraic geometry, elimination theory, and geometric modeling is to determine explicitly the equations defining graphs and images of rational maps.

The school will consist of the following four courses with exercise sessions, plus a Macaulay2 workshop

- Linkage and residual intersections
- Characteristic p methods and applications
- Blowup algebras
- Multiplicity theory

Suggested Prerequisites

- An introduction to Commutative Algebra by Atiyah and MacDonald
- Commutative Algebra with a view towards algebraic geometry by David Eisenbud - Chapters 2-13 and 17-21

Random and arithmetic structures in topology

Dates: June 10 – June 21, 2019

Location: MSRI, Berkeley, CA

Organizers: Alex Furman (University of Illinois at Chicago), Tsachik Gelander (Weizmann Institute of Science)

Description

The study of locally symmetric manifolds, such as closed hyperbolic manifolds, involves geometry of the corresponding symmetric space, topology of towers of its finite covers, and number-theoretic aspects that are relevant to possible constructions.

The summer school will provide an introduction to these and closely related topics such as lattices, invariant random subgroups, and homological methods.

Prerequisites

It will be useful to be familiar with basic notions on:

- Hyperbolic plane H^2 and hyperbolic spaces H^n : disk/ball, upper-half plane/space models, geodesics, isometries
- Elements of Lie groups, in particular $SL(2;R)$ -action on H^2 , and $SO(n;1)$ -action on H^n
- Basic algebraic topology: fundamental group and covering spaces, homology, elements of cohomology, Poincaré duality
- Some notions of geometric group theory: Cayley graphs, quasi-isometries
- Real Analysis: abstract measure theory and integration
- Abstract Algebra: finite extensions of \mathbb{Q} and their Galois groups

Recommended background material

- Hatcher, Algebraic Topology, Chapters 1,2,3
- D.Witte Morris, Introduction to arithmetic groups, Chapters 1, 5, 6. (available at arXiv:0106063)
- Maclachlan, A. Reid, The arithmetic of hyperbolic 3-manifolds, GTM 219. Chapters 1, 2, and possibly 4
- T. Gelander, A lecture on Invariant Random Subgroups. (available at arXiv:1503.08402)

Representation stability

Dates: June 24 – July 05, 2019

Location: MSRI, Berkeley, CA

Organizers: Thomas Church (Stanford University), Andrew Snowden (University of Michigan), Jenny Wilson (Stanford University)

Description

This summer school will give an introduction to representation stability, the study of algebraic structural properties and stability phenomena exhibited by sequences of representations of finite or classical groups -- including sequences arising in connection to hyperplane arrangements, configuration spaces, mapping class groups, arithmetic groups, classical representation theory, Deligne categories, and twisted commutative algebras. Representation stability incorporates tools from commutative algebra, category theory, representation theory, algebraic combinatorics, algebraic geometry, and algebraic topology. This summer school will assume minimal prerequisites, and students in varied disciplines are encouraged to apply.

Suggested Prerequisites

Representations of finite groups over \mathbb{C} and the classification of S_n -irreps

Representation Theory of Finite Groups:

- Fulton--Harris, "Representation Theory, A first course", Chapters 1-3
- Serre, "Linear representations of finite groups", Parts I and II

Representations of S_n :

- Fulton--Harris, "Representation Theory, A first course", Chapter 4
 - James, "The representation theory of symmetric groups"
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Commutative algebra (Noetherian rings, tensor product, free resolutions)

Tensor products:

- Dummit--Foote, "Abstract Algebra", Chapter 10.4
- Atiyah--MacDonald, "Introduction to Commutative Algebra", Chapter 2

Noetherian rings:

- Dummit--Foote, "Abstract Algebra", Chapter 15.1
 - Atiyah--MacDonald, "Introduction to Commutative Algebra", Chapter 6-7
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Gröbner bases

- Dummit--Foote, "Abstract Algebra", Chapter 9.5-9.6
 - Cox--Little--O'Shea, "Ideals, Varieties, and Algorithms", Chapters 2.1-2.6
 - Eisenbud, "Commutative algebra" Chapter 15
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Representation theory of $GL_n\mathbb{C}$

- Henderson, "Representations of Lie Algebras", whole book
 - Fulton--Harris, "Representation Theory, A first course", Chapter 15
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Homological algebra (Tor, Ext, derived functors)

- Dummit--Foote, "Abstract Algebra", Chapter 10.5, 17.1
 - Rotman, "An Introduction to Homological Algebra", Chapter 6.1-6.2 & 7.1-7.2
 - Weibel, "An introduction to homological algebra", Chapters 2, 3
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Topology (homology and cohomology of spaces and/or groups)

Spaces:

- Hatcher, "Algebraic Topology", Chapters 2-3

Groups:

- Brown, "Cohomology of Groups", Chapters I-III
 - Dummit--Foote, "Abstract Algebra", Chapter 17.2
 - Weibel, "An introduction to homological algebra", Chapters 6, 7
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Symmetric functions

- MacDonal, "Symmetric Functions and Hall Polynomials", Chapter I
 - Stanley, "Enumerative Combinatorics, Vol 2", Chapter 7
-

Category theory

- Mac Lane, "Categories for the Working Mathematician", Chapter I.1-I.5, II.1-II.4, & IV.1-IV.4.
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Background articles:

- Church--Ellenberg--Farb, "FI-modules and stability for representations of symmetric groups"
- Sam--Snowden, "Introduction to twisted commutative algebras"
- Draisma, "Noetherian up to symmetry"

Séminaire de Mathématiques Supérieures 2019: Current trends in Symplectic Topology

Dates: July 01 – July 13, 2019

Location: Montreal, Canada

Organizers: Octav Cornea (Université de Montréal), Yakov Eliashberg (Stanford University), Michael Hutchings (University of California, Berkeley), Egor Shelukhin (Université de Montréal)

Description

Symplectic topology is a fast developing branch of geometry that has seen phenomenal growth in the last twenty years. This two weeks long summer school, organized in the setting of the Séminaire de Mathématiques Supérieures, intends to survey some of the key directions of development in the subject today thus covering: advances in homological mirror symmetry; applications to hamiltonian dynamics; persistent homology phenomena; implications of flexibility and the dichotomy flexibility/rigidity; legendrian contact homology; embedded contact homology and four-dimensional holomorphic techniques and others. With the collaboration of many of the top researchers in the field today, the school intends to serve as an introduction and guideline to students and young researchers who are interested in accessing this diverse subject.

Suggested Prerequisites

While a precise list of prerequisites is listed below, some familiarity with the following more specialized, but still accessible, references is a plus:

- D. McDuff, D. Salamon - Introduction to Symplectic Topology, Clarendon Press 1998
- Second half of M.Audin, M.Damian - Morse theory and Floer homology, Springer 2013.

Finally, the technical crux of modern symplectic topology consists in the analysis of moduli spaces of J-holomorphic curves. While this is not a prerequisite to attend this school it is useful to mention here that the basic J-holomorphic machinery is contained in the monograph: 6 D.McDuff, D. Salamon - J-Holomorphic Curves and Symplectic Topology, AMS Monographs, 2004.

Symplectic topology makes use of a variety of techniques from different fields of mathematics and a degree of familiarity with the following is required:

- Basics of manifolds and algebraic topology (such as covered in the book of G. Bredon - Geometry and Topology, Springer 1993);
- Elements of Riemannian geometry (such as the first three chapters of S. Gallot, D. Hulin, J. Lafontaine - Riemannian Geometry, Springer 1987);
- Complex analysis (such as the book of L. Ahlfors - Complex Analysis, McGraw-Hill, 1966);
- The basics of modern Morse theory as covered in the first half of M.Audin, M.Damian - Morse theory and Floer homology, Springer 2013;
- Some elements of Riemann surfaces (such as the first two chapters of H.M.Farkas, I.Kra - Riemann Surfaces, Springer 1980).

Geometric Group Theory

Dates: July 1 – 12, 2019

Location: Oaxaca -- Mexico

Organizers: Rita Jiménez Rolland (Instituto de Matemáticas, UNAM-Oaxaca), Pierre Py (Instituto de Matemáticas, UNAM-Ciudad Universitaria)

Description

Geometric group theory studies discrete groups by understanding the connections between algebraic properties of these groups and topological and geometric properties of the spaces on which they act. The aim of this summer school is to introduce graduate students to specific central topics and recent developments in geometric group theory. The school will also include students presentations to give the participants an opportunity to practice their speaking skills in mathematics. Finally, we hope that this meeting will help connect Latin American students with their American and Canadian counterparts in an environment that encourages discussion and collaboration.

Suggested Prerequisites

Students should have basic knowledge of calculus, mathematical analysis and measure theory (knowing Hilbert space theory and l_p spaces will be enough). They are also expected to know the standard topics in abstract algebra (linear algebra, groups, rings, fields) with an emphasis on group theory. Other relevant topics include differential geometry and topology and algebraic topology (one course on differential geometry and abstract manifolds and knowing the basics of the beginning of Hatcher's book "Algebraic Topology" is enough). Furthermore, having taken a basic course on dynamical systems could be useful (yet this is not mandatory).

Finally, as a general reference on the topics of the school that the students could browse, we mention Drutu and Kapovich's book Geometric Group Theory, specifically chapters 4, 5, 7, 8 and 11 (this book was published by the AMS in 2019, it is available in preprint form from <https://www.math.ucdavis.edu/%7Ekapovich/papers.html>). Further references are Farb and Margalit's book A primer on Mapping Class Group (for instance chapters 1, 2 and 3), the lectures notes Notes on word hyperbolic groups by Alonso, Brady, Cooper, Ferlini, Lustig, Mihalik, Shapiro and Short (in Group theory from a geometrical viewpoint (Trieste, 1990)), the book by Deroin, Navas and Rivas, Groups, Orders and Dynamics (to be published by the AMS, available in preprint form at <https://arxiv.org/abs/1408.5805>), specifically its first chapter and finally the two standard references on group actions on Hilbert spaces (Kazhdan's property (T), by Bekka, de La Harpe and Valette and Groups with the Haagerup property: Gromov's a -T-menability, by Cherix, Cowling, Jolissaint, Julg and Valette). These references would hopefully give a preview of some of the material that will be covered and will help the students to take more advantage of the school.

Polynomial Method

Dates: July 08 – July 19, 2019

Location: MSRI, Berkeley, CA

Organizer: Adam Sheffer (California Institute of Technology), Joshua Zahl (University of British Columbia)

Description

In the past eight years, a number of longstanding open problems in combinatorics were resolved using a new set of algebraic techniques. In this summer school, we will discuss these new techniques as well as some exciting recent developments

Suggested Prerequisites

We intend this summer school to be self-contained. Any student who is familiar with undergraduate level linear algebra and algebra should be able to follow the contents of this course.

Recent topics on well-posedness and stability of incompressible fluid and related topics

Dates: July 22 – August 02, 2019

Location: MSRI, Berkeley, CA

Organizer: Yoshikazu Giga (University of Tokyo), Maria Schonbek (University of California, Santa Cruz), Tsuyoshi Yoneda (University of Tokyo)

Description

The purpose of the summer school is to introduce graduate students to fundamental results on the Navier-Stokes and the Euler equations, with special emphasis on the solvability of its initial value problem with rough initial data as well as the large time behavior of a solution. These topics have long research history. However, recent studies clarify the problems from a broad point of view, not only from analysis but also from detailed studies of orbit of the flow.

General Prerequisites

- H. Brezis,
Functional analysis, Sobolev spaces and partial differential equations.
Springer, New York, 2011.
ISBN: 978-0-387-70913-0
- L. C. Evans,
Partial differential equations. Second edition.
Graduate Studies in Mathematics, 19. American Mathematical Society, Providence, RI, 2010.
ISBN: 978-0-8218-4974-3

Prerequisites by lecturer:

1. Lectures by L. Brandolese

a) J.-Y. Chemin,

Localization in Fourier space and Navier-Stokes system.

Lectures notes of the De Giorgi Center, 2005.

<https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0ahUKEwj8IPnh4rncAhVGrxoKHQYQBccQFggvMAA&url=https%3A%2F%2Fwww.math.uzh.ch%2Fpde13%2Ffileadmin%2Fpde13%2Fpdf%2FcoursPisa.pdf&usg=AOvVaw1RorDZAuT8VSrh-Y7jX0Om>

Phase space analysis of partial differential equations. Vol. I, 53–135,

Pubbl. Cent. Ric. Mat. Ennio Giorgi, Scuola Norm. Sup., Pisa, 2004.

In particular, Chapter 1 and Sections 2.1, 2.2, 2.3.

b) A. J. Chorin and J. E. Marsden,

A mathematical introduction to fluid mechanics. Third edition.

Springer, Corrected fourth printing 2000.

Texts in Applied Mathematics, 4. Springer-Verlag, New York, 1993.

ISBN: 0-387-97918-2

Soft cover :ISBN: 978-1-4612-6934-2

In particular, Chapter 1

2. Lectures by I. Gallagher

a) H. Brezis, Functional analysis, Sobolev spaces and partial differential equations.

Springer, New York, 2011.

ISBN: 978-0-387-70913-0

In particular, Chapters 4, 5, 6, 8, 10

- b) H. Bahouri, J.-Y. Chemin and R. Danchin,
Fourier analysis and nonlinear partial differential equations.
Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], 343.
Springer, Heidelberg, 2011.
ISBN: 978-3-642-16829-1
In particular, Chapter 1 and Section 3.1
- c) L. C. Evans, Partial Differential Equations. Second edition.
Graduate Studies in Mathematics, 19. American Mathematical Society, Providence, RI, 2010.
ISBN: 978-0-8218-4974-3
In particular, Chapters 1, 2, 3, 5
3. Lectures by T. Yoneda
- a) A. Majda and A. Bertozzi,
Vorticity and Incompressible Flow,
Cambridge Texts in Applied Mathematics, 27. Cambridge University Press, Cambridge 2002.
ISBN: 0-521-63057-6; 0-521-63948-4
In particular, Chapter 4
- b) J. Bourgain and D. Li,
Strong ill-posedness of the incompressible Euler equations in borderline Sobolev spaces.
Invent. math. 201, (2015), 97-157.

Toric Varieties in Taipei

Dates: July 29 – August 09, 2019

Location: Taipei

Organizers: David Cox (University of Massachusetts, Amherst), Henry Schenck (Iowa State University)

Description

Toric varieties are algebraic varieties defined by combinatorial data, and there is a wonderful interplay between algebra, combinatorics and geometry involved in their study. Many of the key concepts of abstract algebraic geometry (for example, constructing a variety by gluing affine pieces) have very concrete interpretations in the toric case, making toric varieties an ideal tool for introducing students to abstruse concepts.

Suggested Prerequisites

- Chapters 1,2,3,4,5,8 of "Ideals, Varieties and Algorithms" and Sections 1.0, 2.0, 3.0, 4.0 and 6.0 of "Toric Varieties" (Section 0 of these chapters is a background section that discusses algebraic geometry with no knowledge of toric varieties required). An alternative to the Sections 0 would be my notes "Introduction to Algebraic Geometry", available at my web site <https://dacox.people.amherst.edu/>.
- Chapters 1,2,3,4 of Ravi Vakil's excellent text "Foundation of Algebraic Geometry", freely available at math.stanford.edu/~vakil/216blog/FOAGjun1113public.pdf
- Chapters 1 and 2 of Hartshorne's "Algebraic Geometry".

Mathematics of Machine Learning (Microsoft)

Dates: July 29 – August 09, 2019

Location: Seattle, WA

Organizers: Sebastien Bubeck (Microsoft Research), Anna Karlin (University of Washington), Yuval Peres (University of California, Berkeley), Adith Swaminathan (Microsoft Research)

Description

Learning theory is a rich field at the intersection of statistics, probability, computer science, and optimization. Over the last decades the statistical learning approach has been successfully applied to many problems of great interest, such as bioinformatics, computer vision, speech processing, robotics, and information retrieval. These impressive successes relied crucially on the mathematical foundation of statistical learning.

Recently, deep neural networks have demonstrated stunning empirical results across many applications like vision, natural language processing, and reinforcement learning. The field is now booming with new mathematical problems, and in particular, the challenge of providing theoretical foundations for deep learning techniques is still largely open. On the other hand, learning theory already has a rich history, with many beautiful connections to various areas of mathematics (e.g., probability theory, high dimensional geometry, game theory). The purpose of the summer school is to introduce graduate students (and advanced undergraduates) to these foundational results, as well as to expose them to the new and exciting modern challenges that arise in deep learning and reinforcement learning.

Suggested Prerequisites

- Linear Algebra
(e.g. <https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/>).
- Probability
(e.g. <http://web.archive.org/web/20111203000912/http://www.amazon.com:80/ProbabilityPath-Sidney-Resnick/dp/081764055X>).
- Real Analysis
(e.g. <https://ocw.mit.edu/courses/mathematics/18-100c-real-analysis-fall-2012/>).

H-Principle (INdAM)

Dates: July 29 – August 09, 2019

Location: Cortona, Italy (INdAM)

Organizers: Emmy Murphy (Northwestern University), Giorgio Patrizio (Istituto Nazionale di Alta Matematica "Francesco Severi" (INdAM)), Takashi Tsuboi (University of Tokyo)

Description

This two week summer school will introduce graduate students to the theory of h-principles. After building up the theory from basic smooth topology, we will focus on more recent developments of the theory, particularly applications to symplectic and contact geometry, and foliation theory.

h-principles in smooth topology (Emmy Murphy)

Euler equations and fluid dynamics (Phil Isett)

Contact and symplectic flexibility (Emmy Murphy)

Foliation theory and diffeomorphism groups (Takashi Tsuboi)

Suggested Prerequisites

- Basic Analysis and PDE;
- elementary algebraic topology (homotopy and homology groups).
- elementary differential topology (manifolds, vector fields and differential form calculus, basic Morse theory, vector bundles and characteristic classes).

Please visit www.msri.org/sgs for more information