Summer Graduate Schools 2021

2021 CRM-PIMS Summer School in Probability

Dates: May 24 – June 18, 2021
Location: Centre de Recherches Mathématiques, Montreal, Canada
Organizers: Louigi Addario-Berry (McGill University), Omer Angel (University of British Columbia), Alexander Fribergh (University of Montreal), Mathav Murugan (University of British Columbia), Edwin Perkins (University of British Columbia)

Description
The courses in this summer school focus on mathematical models of group dynamics, how to describe their dynamics and their scaling limits, and the connection to discrete and continuous optimization problems.

The phrase "group dynamics" is used loosely here -- it may refer to species migration, the spread of a virus, or the propagation of electrons through an inhomogeneous medium, to name a few examples. Very commonly, such systems can be described via stochastic processes which approximately behave like the solution of an appropriate partial differential equation in the large-population limit.

Suggested Prerequisites
- A first year-long graduate course in probability, roughly covering the material of Billingsley's Probability and Measure, excuding Sections 9, 28 and 30.
- Rademacher theorem.
- Some familiarity with basic properties of the Ising model and Hamilton-Jacobi equations would be a plus, but not strictly necessary.
    - For the Ising model, the first two chapters of this book: https://www.unige.ch/math/folks/velenik/smbook/
    - For Hamilton-Jacobi equations: Lawrence Evans, Partial Differential Equations, Sections III.10.1 and III.10.2.

Sparsity of Algebraic Points

Dates: June 07 – June 18, 2021
Location: MSRI, Berkeley, CA
Organizers: Philipp Habegger (Universität Basel), Hector Pasten (Pontificia Universidad Católica de Chile)

Description
The theory of Diophantine equations is understood today as the study of algebraic points in algebraic varieties, and it is often the case that algebraic points of arithmetic relevance are expected to be sparse.

This summer school will introduce the participants to two of the main techniques in the subject: (i) the filtration method to prove algebraic degeneracy of integral points by means of the subspace theorem, leading to special cases of conjectures by Bombieri, Lang, and Vojta, and (ii) unlikely intersections through o-minimality and bi-algebraic geometry, leading to results in the context of the Manin-Mumford conjecture, the André-Oort conjecture, and generalizations. This SGS should provide an entry point to a very active research area in modern number theory.

Suggested Prerequisites
Analysis. Standard undergraduate background: Multivariable calculus, holomorphic functions of one complex variable.

Algebra. Standard undergraduate background: Groups, rings, fields, Galois theory.

Logic. No pre-requisites are required. Although the topic of o-minimality pertains to model theory, the necessary model-theoretical background for our diophantine applications is not too demanding, and it will be introduced as needed.

Number theory. Basics of algebraic number theory: number fields, places, finiteness theorems (finiteness of the class group, finite generation of units). Arithmetic of elliptic curves over the rational numbers (e.g. heights, the Mordell-Weil theorem). Classification of complex elliptic curves via the action of SL2(Z) on the upper half plane.

Suggested literature:
- Chapters 2,3,4,5 of Marcus, Number Fields. (UTX, Springer)
- Chapters 1,2,3 of Silverman-Tate, Rational points on Elliptic curves. (UTM, Springer)
- Chapter VII, Sec. 1 and 2 of Serre, A course in Arithmetic. (GTM 7, Springer)

Algebraic geometry. The classical point of view for algebraic varieties over a field is enough (even simply over C). Affine and projective varieties. Divisors, rational functions. Curves (Riemann-Hurwitz, RiemannRoch). Some basic familiarity with abelian varieties, e.g. the Jacobian of a curve over C.

Suggested literature:
- Chapter 1 of Hartshorne, Algebraic Geometry. (GTM 52, Springer)
- Sections A4, A5, A6 of Hindry-Silverman, Diophantine Geometry, an introduction (GTM 201, Springer)

Mathematical Sciences Research Institute

17 Gauss Way   Berkeley, CA 94720-5070   p: 510.642.0143   f: 510.642.8609   www.msri.org

Mathematics of Big Data: Sketching and (Multi-) Linear Algebra

Dates: June 21 – July 02, 2021
Location: MSRI, Berkeley, CA
Organizers: Kenneth Clarkson (IBM Research Division), Lior Horesh (IBM Thomas J. Watson Research Center)

Description
This summer school will introduce graduate students to sketching-based approaches to computational linear and multi-linear algebra. Sketching here refers to a set of techniques for compressing a matrix, to one with fewer rows, or columns, or entries, usually via various kinds of random linear maps. We will discuss matrix computations, tensor algebras, and such sketching techniques, together with their applications and analysis.

Suggested Prerequisites
The minimum requirement for students to beneficially participate in this summer schools are the basics of probability, algorithms, linear algebra.

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

Foundations and Frontiers of Probabilistic Proofs

Dates: June 28 – July 09, 2021
Location: ETH Zurich, Switzerland
Organizers: Alessandro Chiesa (University of California, Berkeley), Tom Gur (University of Warwick)

Description
Proofs are at the foundations of mathematics. Viewed through the lens of theoretical computer science, verifying the correctness of a mathematical proof is a fundamental computational task. Indeed, the P versus NP problem, which deals precisely with the complexity of proof verification, is one of the most important open problems in all of mathematics.

The complexity-theoretic study of proof verification has led to exciting reenvisionings of mathematical proofs. For example, probabilistically checkable proofs (PCPs) admit local-to-global structure that allows verifying a proof by reading only a minuscule portion of it. As another example, interactive proofs allow for verification via a conversation between a prover and a verifier, instead of the traditional static sequence of logical statements. The study of such proof systems has drawn upon deep mathematical tools to derive numerous applications to the theory of computation and beyond.

In recent years, such probabilistic proofs received much attention due to a new motivation, delegation of computation, which is the emphasis of this summer school. This paradigm admits ultra-fast protocols that allow one party to check the correctness of the computation performed by another, untrusted, party. These protocols have even been realized within recently-deployed technology, for example, as part of cryptographic constructions known as succinct non-interactive arguments of knowledge (SNARKs).

This summer school will provide an introduction to the field of probabilistic proofs and the beautiful mathematics behind it, as well as prepare students for conducting cutting-edge research in this area.

Courses. The summer graduate school will start with two days of introductory material for everyone, covering definitions and simple examples of interactive proofs and probabilistically checkable proofs. Subsequently, two complementary courses will be offered:

- A *foundations course*, covering the "classics" of probabilistic proofs. The material includes seminal results that have found a diverse set of applications in theoretical computer science.
- A *frontiers course*, covering contemporary results in probabilistic proofs. The focus of this course is on proof protocols for delegating computations.

Students are encouraged to attend both courses, which have comparable difficulty and are designed to complement each other.

Suggested Prerequisites

(1) Fundamentals of computational complexity.

For example, chapters 1,2,4,6,7 of Computational Complexity: A Modern Approach by Sanjeev Arora and Boaz Barak. Most importantly:

- Turing machines, complexity classes, and reductions (1.2-1.5, 2.2).
- The Cook-Levin theorem (2.3).
- Familiarity with the classes P, NP, PSPACE, NEXP, and their complete languages (1.6, 2.1, 2.6, 4.1, 4.2).
- The computation model of Boolean circuits, circuit satisfiability, and exponential size circuits (6.1-6.4, 6.8).
- Probabilistic computation and the class BPP (7.1-7.4).

(2) Basic knowledge of finite fields and their properties.

For example, as covered in the following references.

- Introduction to finite fields.
- A. Sutherland's notes on finite fields and integer arithmetic.
- V. Guruswami's cheat sheet on finite fields.

For example, as covered in A. Sutherland's notes on finite fields and integer arithmetic.

MSRI
Mathematical Sciences Research Institute

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

Metric Geometry and Geometric Analysis

Dates: July 05 – July 16, 2021
Location: University of Oxford, United Kingdom
Organizers: Cornelia Drutu (University of Oxford), Panos Papazoglou (University of Oxford)

Description
The purpose of the summer school is to introduce graduate students to key mainstream directions in the recent development of geometry, which sprang from Riemannian Geometry in an attempt to use its methods in various contexts of non-smooth geometry. This concerns recent developments in metric generalizations of the theory of nonpositively curved spaces and discretizations of methods in geometry, geometric measure theory and global analysis. The metric geometry perspective gave rise to new results and problems in Riemannian Geometry as well.

All these themes are intertwined and have developed either together or greatly influencing one another. The summer school will introduce some of the latest developments and the remaining open problems in these very modern areas, and will emphasize their synergy.

Suggested prerequisites
We anticipate that the participating students will come from diverse backgrounds in Geometric Group Theory, Riemannian Geometry and Geometric Analysis.

The following textbooks would give adequate preparation but we don't expect students to have necessarily studied all of them:

- Chapters 3, 8, 10, 11 of "Geometric Group Theory" by C. Drutu and M. Kapovich.
- Sections 2,7,8 of "A Course in Metric Geometry" by D. Burago, Y. Burago and M. S. Ivanov.
- the first chapters of "Geometric Measure Theory" by F. Morgan.
- Parts I.2, I.3, I.8, and II.1 of "Metric Spaces of Non-Positive Curvature" by Bridson-Haefliger.

The students who have been selected to take part in the school will be sent a compilation of introductory texts extracted from the books above and other existing lecture notes, for study beforehand, by email and, if they so wish, a paper version by ordinary mail as well. A more definite and specific list of prerequisites will be given later on after consulting with the lecturers so that background material matches the specific topics they plan to cover in their lectures.

MSRI
Mathematical Sciences Research Institute

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

Gauge Theory in Geometry and Topology

Dates: July 05 – July 16, 2021
Location: MSRI, Berkeley, CA
Organizers: Lynn Heller (Universität Hannover), Francesco Lin (Columbia University), Laura Starkston (University of California, Davis), Boyu Zhang (Princeton University)

Description
Gauge theory is a geometric language used to formulate many fundamental physical phenomena, which has also had profound impact on our understanding of topology. The main idea is to study the space of solutions to partial differential equations admitting a very large group of local symmetries. Starting in the late 1970s, mathematicians began to unravel surprising connections between gauge theory and many aspects of geometric analysis, algebraic geometry and low-dimensional topology. This influence of gauge theory in geometry and topology is pervasive nowadays, and new developments continue to emerge.

The goal of the summer school is to introduce students to the foundational aspects of gauge theory, and explore their relations to geometric analysis and low-dimensional topology. By the end of the two-week program, the students will understand the relevant analytic and geometric aspects of several partial differential equations of current interest (including the Yang-Mills ASD equations, the Seiberg-Witten equations, and the Hitchin equations) and some of their most impactful applications to problems in geometry and topology.

Suggested prerequisites
We ask the graduate student participants to study the following topics either in course work or self-study in advance in order to arrive at the summer school well prepared. The essential material regarding the last three topics will be reviewed in the first few lectures of the summer school.

Differential Topology: Smooth manifolds, transversality, and Lie groups. References for these topics include:
- Guillemin-Pollack, Differential Topology (Chapters 1-2)
- Milnor, Topology from a Differentiable Viewpoint (Chapters 1-2)
- Warner, Foundations of Differentiable Manifolds and Lie Groups (Chapters 1-4)

Algebraic topology: Singular and CW Homology:
- Hatcher Algebraic Topology (Chapters 2,3)

Functional analysis:
- Donaldson-Kronheimer, The geometry of 4-manifolds (Appendix)
- Evans, Partial Differential Equations (Appendix B)

Riemannian geometry and bundles:
- Roe, Elliptic operators, topology and asymptotic methods (Chapters 1-2)

Riemann surfaces:
- Donaldson, Riemann surfaces (Chapters 1-4)

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

Random Conformal Geometry

Dates: July 19 – July 30, 2021
Location: MSRI, Berkeley, CA
Organizers: Mario Bonk (University of California, Los Angeles), Steffen Rohde (University of Washington), Fredrik Viklund (Royal Institute of Technology)

Description
This Summer Graduate School will cover basic tools that are instrumental in Random Conformal Geometry (the investigation of analytic and geometric objects that arise from natural probabilistic constructions, often motivated by models in mathematical physics) and are at the foundation of the subsequent semester-long program "The Analysis and Geometry of Random Spaces". Specific topics are Conformal Field Theory, Brownian Loops and related processes, Quasiconformal Maps, as well as Loewner Energy and Teichmüller Theory.

Suggested Prerequisites
We expect that the students have some solid knowledge of real and complex analysis corresponding to basic first-year graduate courses (as presented in the first 16 Chapters of Rudin's "Real and Complex Analysis", for example). The students should also have some background in basic probability up to the central limit theorem (corresponding to Chapters 1-3 in Durrett's "Probability"), but we do not expect knowledge of more advanced topics such as Brownian motion. z

MSRI
Mathematical Sciences Research Institute

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

Recent Topics in Well Posedness

Dates: July 19 – July 30, 2021
Location: National Center For Theoretical, Taipei, Taiwan
Organizers: Jungkai Chen (National Taiwan University), Yoshikazu Giga (University of Tokyo), Maria Schonbek (University of California, Santa Cruz), Tsuyoshi Yoneda (University of Tokyo)

Description
The purpose of the workshop is to introduce graduate students to fundamental results on the Navier-Stokes and the Euler equations, with special emphasis on the solvability of its initial value problem with rough initial data as well as the large time behavior of a solution. These topics have long research history. However, recent studies clarify the problems from a broad point of view, not only from analysis but also from detailed studies of orbit of the flow.

Suggested Prerequisites

- H. Brezis,
  Functional analysis, Sobolev spaces and partial differential equations.
  Springer, New York, 2011.
  ISBN: 978-0-387-70913-0
- L. C. Evans,
  Partial differential equations. Second edition.
  Graduate Studies in Mathematics, 19. American Mathematical Society, Providence, RI, 2010.
  ISBN: 978-0-8218-4974-3

Please visit www.msri.org/sgs for more information